Elliptic Genus of E-strings

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Based on arXiv:1411.2324
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6d SCFTs

- Non-trivial CFTs of the highest dimension.
  - Worldvolume theory living on branes
  - Compactification of string theory

- (2,0) SCFT has been studied over the last few years. [Witten]
- (1,0) SCFT with E_8 global symmetry [Ganor, Hanany], [Seiberg, Witten]
  - A single M5-brane probing the M9-brane.
  - No Lagrangian description has been known.

$$(B_{\mu\nu},\ \Phi,\ \text{fermions})$$

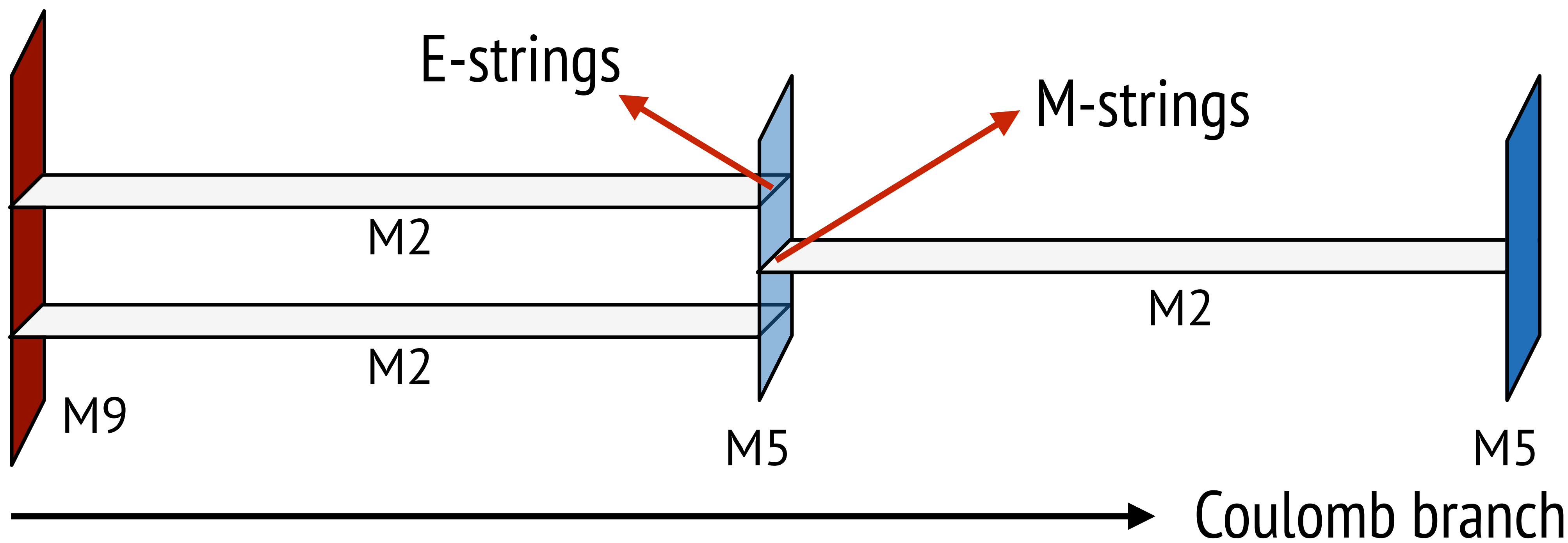
$$H = *H \text{ for } H = dB$$

VEVs determine the location of M5's

  - Location of M5's parametrizes the Coulomb branch.

Self-dual strings

- M2-branes suspended between M5-M9, M5-M5 branes.
  - Stringy excitations coupled to the self-dual tensor.



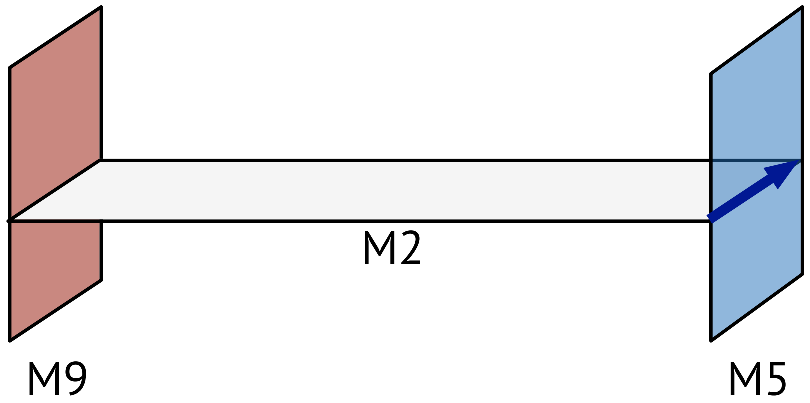
- Compactify the shared dimension among M2 and M5 branes.
- E-string indices → 6d SCFT index in the Coulomb branch

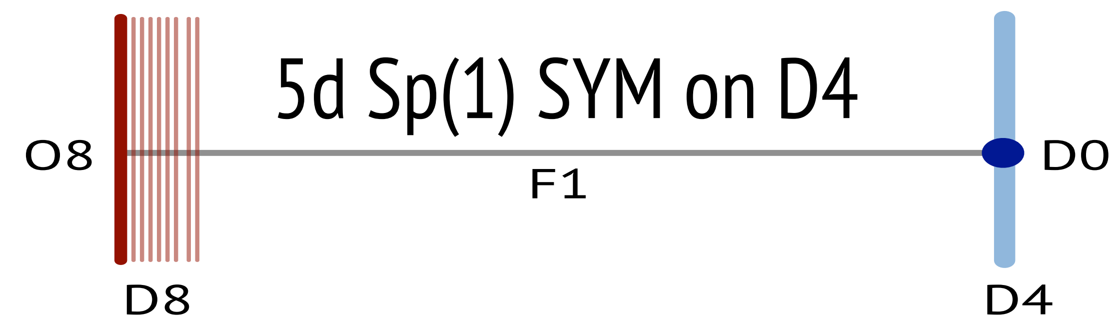
$$Z^{6\mathrm{d}} = 1 + \sum_{n=1}^{\infty} w^n Z_n^{\mathrm{E}}$$

- Find the weakly-coupled description for E-strings.

5d super Yang-Mills

- Reduce the circle shared among M9, M5, M2-branes.





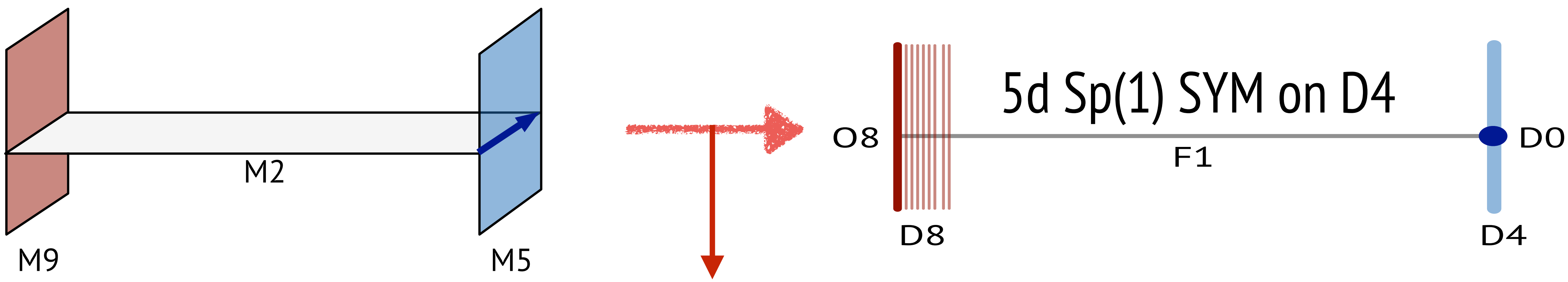
E_8 Wilson line breaking E_8 into SO(16): $RA = (0,0,0,0,0,0,0,1)$

Before	After
M5	D4
M2	F1
M9	O8 + 8 D8
E-strings	W-bosons
Momentums	Instanton solitons

5d super Yang-Mills

[H.-C. Kim, S. Kim, E. Koh, K. Lee, S. Lee] for M-strings
[C. Hwang, JK, S. Kim, J. Park]

- Reduce the circle shared among M9, M5, M2-branes.



E_8 Wilson line breaking E_8 into SO(16): $RA = (0,0,0,0,0,0,0,1)$

- Instanton partition function of 5d SYM
  - counts the bound states of W-bosons and instantons.
  - can be interpreted as 6d SCFT index

$$Z^{6\mathrm{d}} = 1 + \sum_{n=1}^{\infty} w^n Z_n^{\mathrm{E}}(q) = 1 + \sum_{n=1}^{\infty} q^n Z_n^{\mathrm{inst}}(w)$$

  - displays $SO(16) \subset E_8$ symmetry only.

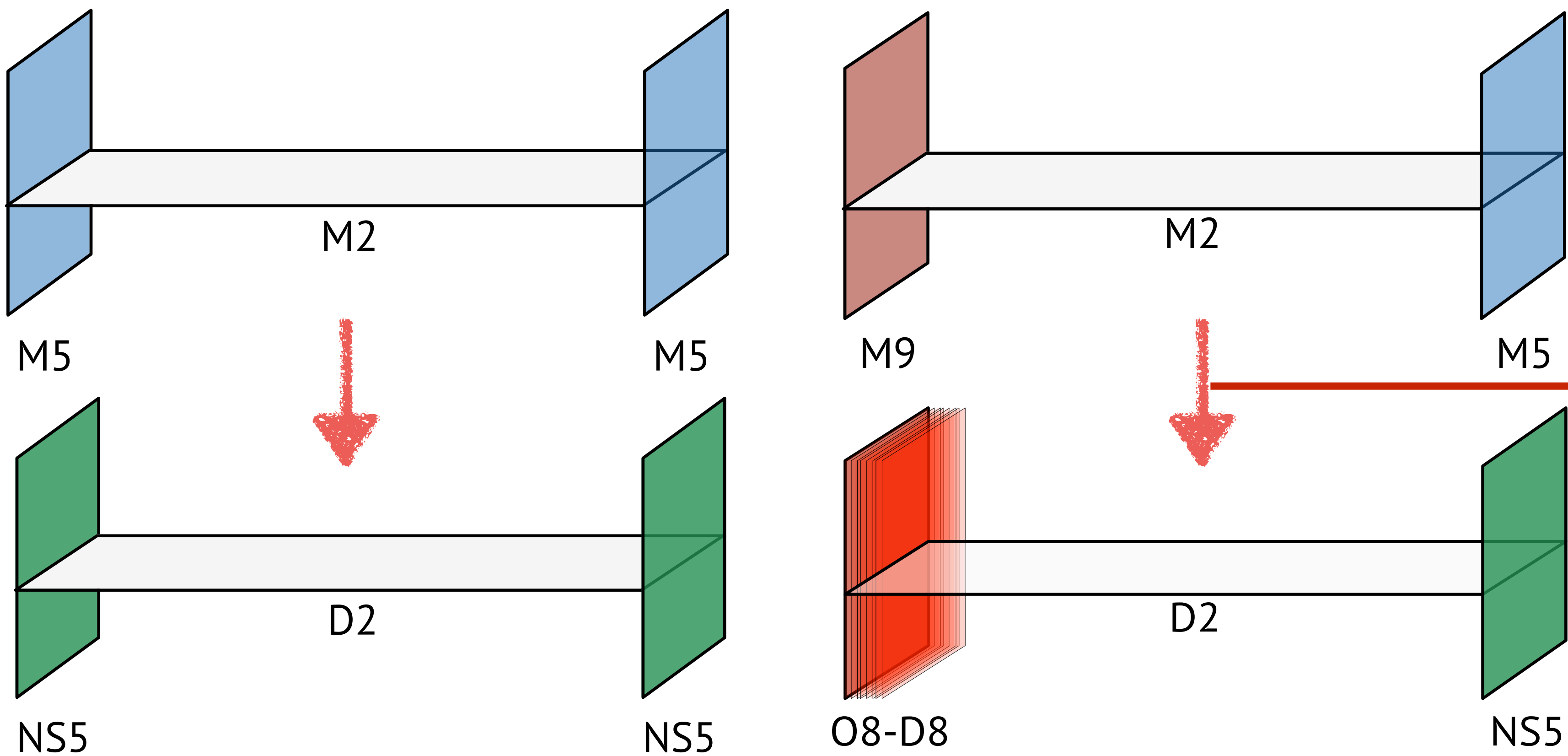
2d gauge theory

	0	1	2	3	4	5	6	7	8	9	10
M5	•	•		•	•	•	•				
M9	•	•		•	•	•	•	•	•	•	•
M2	•	•	•								

R^4 $\quad$ $R^3 \times S^1$

$SO(4)_1 \times SO(3)_2$ symmetry in UV

- Reduce the circle transverse to M5 and M2 branes.



E_8 Wilson line breaking E_8 into SO(16)

$$RA = (0,0,0,0,0,0,0,1)$$

- Strong coupling limit = decompactification of the M-circle.

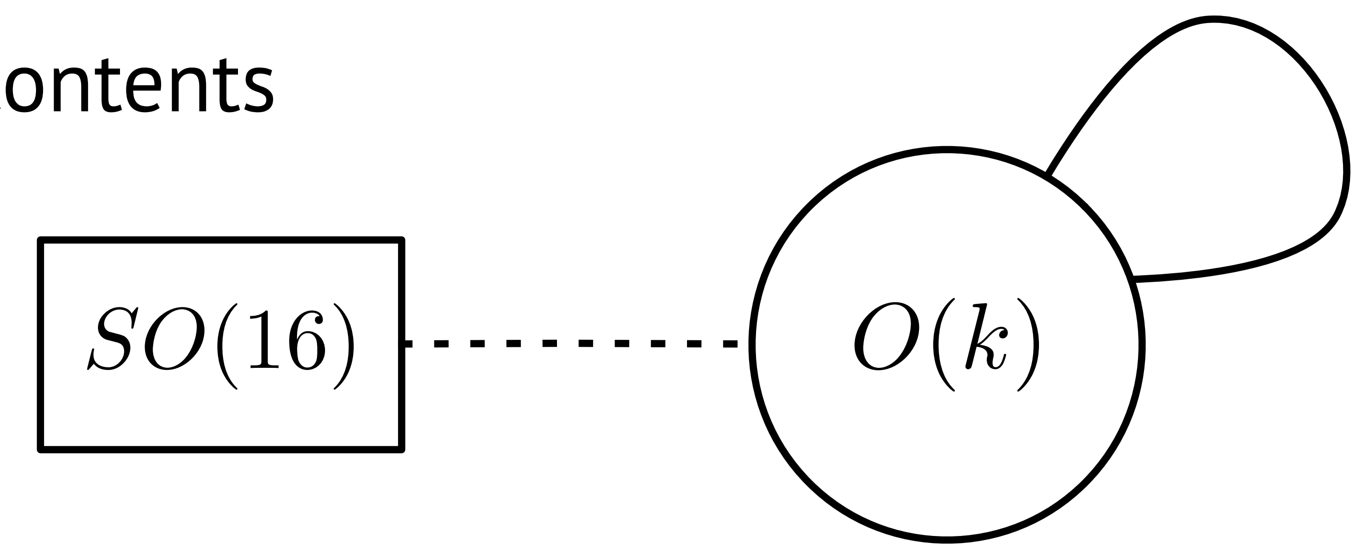
$$g_{\mathrm{YM}}^2 \propto g_s \propto R_{10}$$

  - Full E_8 symmetry & $SO(4)_2$ should be restored.

2d gauge theory

- We consider 2d O(k) gauge theory on the D2-branes.
  - (0,4) supersymmetry $\longrightarrow Q^A_{\dot\alpha}$
  - SU(2)$_{1L}$ X SU(2)$_{1R}$ X SU(2)$_2$ $\longrightarrow J_1, J_2, J_I \longleftarrow \alpha, \dot\alpha, A$
  - SO(16) flavor symmetry $\longrightarrow F_{i=1,\cdots,8}$

- Field contents



$\varphi_{\alpha\dot\beta},\ \lambda^{\alpha A}_-$

O(k) symmetric hyper (D2-D2)

Ψ_l

O(k) X SO(16) Fermi (D2-D8)

$A_\mu,\ \lambda^{\dot\alpha A}_+$

O(k) adjoint vector (D2-D2)

Elliptic genus computation

- The index for k E-strings is defined as

$$Z_k = \mathrm{Tr}_{\mathrm{RR}}\left[(-1)^F q^{H_L}\bar{q}^{H_R} e^{2\pi i\epsilon_1(J_1+J_I)} e^{2\pi i\epsilon_2(J_2+J_I)}\prod_{l=1}^{8} e^{2\pi i m_l F_l}\right]$$

$\sim \{Q^1_{\dot 1}, Q^2_{\dot 2}\}$ SO(4) rotation $SU(2)_2$ SO(16) flavor

  - (0,2) supersymmetry is used to define the index.
  - Path-integral localization: [Benini, Eager, Hori, Tachikawa]

- Zero-modes: O(k) holonomies on the torus. $u = \oint dt\, A_t + \tau \oint ds\, A_s$
  - Many disconnected sectors can appear.

$$\begin{aligned}
&(\mathrm{ee}) : \mathrm{diag}(e^{iu_{1i}\sigma_2})_p,\ \mathrm{diag}(e^{iu_{2i}\sigma_2})_p\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1,-1,1)_{p-2},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,1,-1,-1)_{p-2}\\
&(\mathrm{eo}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},1,1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1)_{p-1}\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},-1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1)_{p-1}\\
&(\mathrm{oe}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,1)_{p-1}\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},-1,-1)_{p-1}\\
&(\mathrm{oo}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1)_{p-1}\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},-1,1)_{p-1}
\end{aligned}$$

for k = 2p

$$\begin{aligned}
&(\mathrm{ee}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},1)_p,\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1)_p\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},-1,-1,1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1,-1)_{p-1}\\
&(\mathrm{eo}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},1)_p,\ \mathrm{diag}(e^{iu_{2i}\sigma_2},-1)_p\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},-1,-1,1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1,1)_{p-1}\\
&(\mathrm{oe}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},-1)_p,\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1)_p\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},1,-1,1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},-1,-1,1)_{p-1}\\
&(\mathrm{oo}) : \mathrm{diag}(e^{iu_{1i}\sigma_2},-1)_p,\ \mathrm{diag}(e^{iu_{2i}\sigma_2},-1)_p\ ;\ \ \mathrm{diag}(e^{iu_{1i}\sigma_2},1,1,-1)_{p-1},\ \mathrm{diag}(e^{iu_{2i}\sigma_2},1,-1,1)_{p-1}
\end{aligned}$$

for k = 2p+1

Elliptic genus computation & Result

- Integrating out non-zero modes: 1-loop determinants.

$$Z_{\text{vector}} = \left(2\pi\eta^2\right)^r \cdot \prod_{\alpha\in\text{root}} \frac{\theta_1(\alpha(u))\theta_1(\epsilon_1+\epsilon_2+\alpha(u))}{\eta^2} \cdot \prod_{a=1}^{r} du_a$$

$$Z_{\text{sym. hyper}} = \prod_{\rho\in\text{sym}} \frac{\eta^2}{\theta_1(\epsilon_1+\rho(u))\theta_1(\epsilon_2+\rho(u))} \qquad Z_{SO(16)\ \text{Fermi}} = \prod_{\rho\in\text{fund}}\prod_{l=1}^{8} \frac{\theta_1(m_l+\rho(u))}{\eta}$$

- The remaining zero-mode integral $Z_k = \sum_a \frac{1}{|W_a|} \cdot \frac{1}{(2\pi i)^{r_a}} \oint Z_{\text{1-loop},a}$
    - Correct contour is specified by the Jeffrey-Kirwan residues. [Benini, Eager, Tachikawa, Hori]

E$_8$ theta function

- 1 E-string result: $Z_1 = \sum_{n=1}^{4} \frac{1}{2} Z_{1(n)} = -\frac{\eta^{-6}}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \cdot \boxed{\frac{1}{2}\sum_{n=1}^{4}\prod_{l=1}^{8}\theta_n(m_l)}$
    - It matches with the result obtained by [Klemm, Mayr, Vafa]

More results

- 2 E-strings:

$$Z_{2(0)} = \frac{1}{2\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)}\sum_{i=1}^{4}\left[\frac{\prod_{l=1}^{8}\theta_i(m_l\pm\frac{\epsilon_1}{2})}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2-\epsilon_1)}+\frac{\prod_{l=1}^{8}\theta_i(m_l\pm\frac{\epsilon_2}{2})}{\theta_1(2\epsilon_2)\theta_1(\epsilon_1-\epsilon_2)}\right]$$

$$Z_{2(1)} = \frac{\theta_2(0)\theta_2(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_2(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}\ ,\quad Z_{2(2)} = \frac{\theta_2(0)\theta_2(2\epsilon_+)\prod_{l=1}^{8}\theta_3(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}$$

$$Z_{2(3)} = \frac{\theta_4(0)\theta_4(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}\ ,\quad Z_{2(4)} = \frac{\theta_4(0)\theta_4(2\epsilon_+)\prod_{l=1}^{8}\theta_2(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}$$

$$Z_{2(5)} = \frac{\theta_3(0)\theta_3(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}\ ,\quad Z_{2(6)} = \frac{\theta_3(0)\theta_3(2\epsilon_+)\prod_{l=1}^{8}\theta_2(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}$$

- 3 E-strings:

$$\begin{aligned} Z_{3(i)} \;=\; & -\frac{\eta^4}{\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2}\left[\frac{\eta^2\theta_1(\epsilon_1)\theta_1(\epsilon_2)}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2-\epsilon_1)\theta_1(3\epsilon_1)\theta_1(\epsilon_2-2\epsilon_1)}\prod_{l=1}^{8}\frac{\theta_i(m_l)\theta_i(m_l\pm\epsilon_1)}{\eta^3}\right. \\ & \left.+\frac{1}{2}\sum_{a=1}^{4}\frac{\eta^2\theta_{\sigma_i(a)}(\frac{3\epsilon_1}{2}+\epsilon_2)\theta_{\sigma_i(a)}(-\frac{\epsilon_1}{2})}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2-\epsilon_1)\theta_{\sigma_i(a)}(\frac{3\epsilon_1}{2})\theta_{\sigma_i(a)}(\epsilon_2-\frac{\epsilon_1}{2})}\prod_{l=1}^{8}\frac{\theta_i(m_l)\theta_a(m_l\pm\frac{\epsilon_1}{2})}{\eta^3}+(\epsilon_1\leftrightarrow\epsilon_2)\right] \end{aligned}$$

$$Z_{3(1)'} = -\eta^6\frac{\theta_2(0)\,\theta_3(0)\,\theta_4(0)}{\theta_1(\epsilon_1)^3\,\theta_1(\epsilon_2)^3}\frac{\theta_2(2\epsilon_+)\,\theta_3(2\epsilon_+)\,\theta_4(2\epsilon_+)}{\theta_2(\epsilon_1)\,\theta_2(\epsilon_2)\,\theta_3(\epsilon_1)\,\theta_3(\epsilon_2)\,\theta_4(\epsilon_1)\,\theta_4(\epsilon_2)}\prod_{l=1}^{8}\frac{\theta_2(m_l)\,\theta_3(m_l)\,\theta_4(m_l)}{\eta^3}$$

$$Z_{3(2)'} = -\eta^6\frac{\theta_2(0)\,\theta_3(0)\,\theta_4(0)}{\theta_1(\epsilon_1)^3\,\theta_1(\epsilon_2)^3}\frac{\theta_2(2\epsilon_+)\,\theta_3(2\epsilon_+)\,\theta_4(2\epsilon_+)}{\theta_2(\epsilon_1)\,\theta_2(\epsilon_2)\,\theta_3(\epsilon_1)\,\theta_3(\epsilon_2)\,\theta_4(\epsilon_1)\,\theta_4(\epsilon_2)}\prod_{l=1}^{8}\frac{\theta_1(m_l)\,\theta_2(m_l)\,\theta_3(m_l)}{\eta^3}$$

$$Z_{3(3)'} = -\eta^6\frac{\theta_2(0)\,\theta_3(0)\,\theta_4(0)}{\theta_1(\epsilon_1)^3\,\theta_1(\epsilon_2)^3}\frac{\theta_2(2\epsilon_+)\,\theta_3(2\epsilon_+)\,\theta_4(2\epsilon_+)}{\theta_2(\epsilon_1)\,\theta_2(\epsilon_2)\,\theta_3(\epsilon_1)\,\theta_3(\epsilon_2)\,\theta_4(\epsilon_1)\,\theta_4(\epsilon_2)}\prod_{l=1}^{8}\frac{\theta_1(m_l)\,\theta_3(m_l)\,\theta_4(m_l)}{\eta^3}$$

$$Z_{3(4)'} = -\eta^6\frac{\theta_2(0)\,\theta_3(0)\,\theta_4(0)}{\theta_1(\epsilon_1)^3\,\theta_1(\epsilon_2)^3}\frac{\theta_2(2\epsilon_+)\,\theta_3(2\epsilon_+)\,\theta_4(2\epsilon_+)}{\theta_2(\epsilon_1)\,\theta_2(\epsilon_2)\,\theta_3(\epsilon_1)\,\theta_3(\epsilon_2)\,\theta_4(\epsilon_1)\,\theta_4(\epsilon_2)}\prod_{l=1}^{8}\frac{\theta_1(m_l)\,\theta_2(m_l)\,\theta_4(m_l)}{\eta^3}$$

- 4 E-strings

- Our results are tested against various literatures.
  - Full elliptic genus for 1, 2 E-strings [Klemm, Mayr, Vafa], [Haghighat, Lockhart, Vafa]
  - Unrefined elliptic genus for 4 E-strings [Sakai]
  - Topological string amplitudes [Mohri], [Huang, Klemm, Poretschkin]
  - 5d Sp(1) instanton partition function [C. Hwang, JK, S. Kim, J. Park]

Summary

- When a single M5-brane probes the M9-brane, 6d (1,0) CFT living on the M5-brane captures the BPS self-dual strings, 'E-strings', which are M2-branes suspended between M5-M9.

- From the string duality, we identified gauge theories living on D2-branes, suspended between 1 NS5- & 8 D8-branes on an O8-plane, as the weakly-coupled description of E-strings.

- We computed the elliptic genus of E-strings using this gauge theories, and tested against the known results, from 5d Sp(1) instanton calculus as well as topological strings.

- This approach allows us to compute the elliptic genus for arbitrary number of E-strings.